EVALUATING WEAPONS SYSTEM USING FUZZY APPROXIMATE REASONING

Mahmod Othman, Ku Ruhana Ku-Mahamud, Mohamad Fadhilli Yahaya

Abstract:
This paper proposed the application of the fuzzy evaluation model using fuzzy sets and approximate reasoning. The objective of the study is to evaluate the weapon system in a subjective environment. The proposed method based on fuzzy sets has initiated the idea of membership set score value evaluation of each criterion alternative. This enables the inclusion of requirements which are incomplete and imprecise. The approximate reasoning of the method allows the decision maker to make the best choice in accordance to human thinking and reasoning processes. The proposed model is based on fuzzy multi-criteria decision-making that consists of fuzzy rules. The use of fuzzy rules, which are extracted directly from input data in making evaluation, contributes to a better decision in selecting the best option and less dependency on the domain of experts. Finally, we constructed a practical example for evaluating attack helicopters to demonstrate the proposed method. From these results, the proposed method shows outstanding performance in comparing with Cheng et al.’s method with 100% accuracy in ranking three attack helicopter alternatives, S1 (MI-28), S2 (AH-64, and S3 (AH-1w). Again the subjective evaluation method showed the advantages of simpler rules properties in NR, Max_L and Min_L. This research work has achieved its objective and produced good evaluation results. This portrays its major advantages in making decisions in new cases, where there is limited or an absence of specific knowledge.

Keywords: Fuzzy sets, Multi-criteria decision making, approximate reasoning.

1. INTRODUCTION

In making good decision, a highly reliable evaluation method is crucial. Such evaluation normally conducted using subjective criteria requires one to use his or her wisdom, experience, professional knowledge and information, which is difficult to define or describe precisely. When analysing using incomplete data, a lot of uncertainties will arise this will confuse decision-makers and will complicate decision-making as it is made under unknown situations. The application of fuzzy sets theory in evaluation systems can improve the evaluation results (Turban, et al. 2000). Several researchers have tried to solve this problem through analytical hierarchy process (AHP), for example in personnel selection (Liang & Wang, 1992; Sonja, 2001) and shipping performance evaluation (Chou & Liang, 2001), whereby evaluation was done by aggregating all the fuzzy sets. However, the presence of
imprecision, vagueness and subjectivity at each level further accumulates greatly the undesired elements in aggregating the marks.

In the literature, various concepts have been proposed focusing on the combination of fuzzy logic model with multi objective decision that can assist in reducing errors in deriving to a judgment (Pedrycz & Gomide, 1982; Liang & Wang, 1992). The research provides approaches to judgment procedure on personnel selection through the development of AHP fuzzy multi criteria. It is cited as being able to minimise subjectivity. Some research in fuzzy evaluation methods is discussed in Othman (2004a; 2004b; 2004c). The authors have proposed algorithms based either on fuzzy similarity function or fuzzy synthetic decision and ranking procedure through satisfaction function. Fuzzy sets membership enables the interpretations of linguistic variables in a very natural and plausible way to formulate and solve various problems. However, expressing the linguistic variable using the singleton fuzzy sets such as in Capaldo and Weon (2001) could result in the loss of much important information and would additionally complicate the course of action. Although many evaluation methods for selecting or ranking have been suggested in the literature, there is yet a method which can give a satisfactory solution to every situation. For this reason, a fuzzy evaluation method is proposed by combining the concepts introduced by Othman (2003) and integrating it with a fuzzy rule (Othman, 2004a) that is derived automatically from input data. This research makes its contribution by introducing the bridging and linking of these two methods. Previous studies on fuzzy evaluation methods evaluate (Tseng & Yeh, 2000; Chang & Yeh, 2002; and Kuo & Chen, 2002) the use of the number of respondents who answered the survey questions to represent fuzzy set in the forms of membership function. However, these methods have a drawback, whereby they are unable to produce a generalised fuzzy evaluation method to evaluate various types of data. Hence, this research introduced the membership set score where various input data are transformed and are not predetermined by the expert. This is important to ensure the consistency in generalising the proposed framework.

The paper proceeds as follows. The proposed model is introduced in Section 2. Section 4 presents the algorithm of the proposed model and numerical results. It is followed by the concluding in Section 5.

2.0 Case Background

Data on the evaluation attack helicopters based on linguistic variable (Cheng et al., 1999) has been used. The five criteria for the best attack helicopter namely technological advance, logistic capability, armament, avionics, and subsisting ability. Three alternatives labeled as S1 (MI-28), S2 (AH-64), and S3 (AH-1w) are used to decide entities of the attack helicopters. Table 1 depicted the data of technology advances for three attack helicopters and their judgement criteria. This table contained the original data of performance scores given by the expert’s opinion on particular subcriteria. From these scores the membership functions are constructed and tabulated in the last column of the table. Linguistic terms are found to be easy to express the subjective and imprecise assessments (Yeh et al., 2000). The membership functions are characterized by the triangular fuzzy number defined as triplet \((a_1, a_2, a_3)\). The triangular fuzzy numbers are the average performance rating values in range of \(a_1\) and \(a_3\). The membership function is defined as

\[
\mu_A(x) = \begin{cases} 
0 & x_i < a_1 \\
\frac{x_i - a_1}{T - a_1} & a_1 \leq x_i \leq T \\
\frac{T - x_i}{T - a_3} & x_i \geq T
\end{cases}
\]

where, \(x_i\) is fuzzy evaluation of alternative in term of triangular fuzzy number, \(T\) is the vertex of the triangular fuzzy number and \(a_1\) and \(a_3\) \(x_i \leq a_1\) two endpoints.
Table 1: data of technology advances for three attack helicopters

<table>
<thead>
<tr>
<th>Item</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>Membership function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Turbo-shafts (kW)</td>
<td>1633 x 2</td>
<td>1265 x 2</td>
<td>1285 x 2</td>
<td>(\mu_m = \begin{cases} \frac{x-1100}{400}, &amp; 1100 \leq x \leq 1500 \ 1, &amp; 1500 \leq x \end{cases})</td>
</tr>
<tr>
<td>2 Weight empty (kg)</td>
<td>7000</td>
<td>5092</td>
<td>4634</td>
<td>(\mu_m = \begin{cases} \frac{8000-x}{4000}, &amp; 4000 \leq x \leq 8000 \ 0, &amp; 8000 \leq x \end{cases})</td>
</tr>
<tr>
<td>3 Max level speed (km/h)</td>
<td>300</td>
<td>293</td>
<td>282</td>
<td>(\mu_m = \begin{cases} \frac{x-250}{70}, &amp; 250 \leq x \leq 320 \ 1, &amp; 320 \leq x \end{cases})</td>
</tr>
<tr>
<td>4 Max disc loading (kg/m^2)</td>
<td>49</td>
<td>56.69</td>
<td>39.80</td>
<td>(\mu_m = \begin{cases} \frac{x-30}{40}, &amp; 30 \leq x \leq 70 \ 1, &amp; 70 \leq x \end{cases})</td>
</tr>
<tr>
<td>5 Service ceiling (m)</td>
<td>5800</td>
<td>6400</td>
<td>4270</td>
<td>(\mu_m = \begin{cases} \frac{x-4000}{3000}, &amp; 4000 \leq x \leq 7000 \ 0, &amp; 7000 \leq x \end{cases})</td>
</tr>
<tr>
<td>6 Maximal range standard fuel (km)</td>
<td>460</td>
<td>482</td>
<td>507</td>
<td>(\mu_m = \begin{cases} \frac{x-400}{200}, &amp; 400 \leq x \leq 600 \ 1, &amp; 600 \leq x \end{cases})</td>
</tr>
<tr>
<td>7 Endurance with maximum fuel</td>
<td>2h</td>
<td>3h 9min</td>
<td>2h</td>
<td>(\mu_m = \begin{cases} 0.5, &amp; 2h \leq x \leq 3h \ 1, &amp; 3h \leq x \end{cases})</td>
</tr>
<tr>
<td>8 g-limits</td>
<td>+3/ - 0.5</td>
<td>+3.5/ - 0.5</td>
<td>+2.5/ - 0.5</td>
<td>(\mu_{gl} = \begin{cases} \frac{(x^+ - 2)}{2}, &amp; 2 \leq x^+ \leq 4 \ 1, &amp; 4 \leq x^+ \end{cases})</td>
</tr>
<tr>
<td>9 Mission capable</td>
<td>90</td>
<td>93</td>
<td>92</td>
<td>(\mu_{mc} = \begin{cases} \frac{x}{100}, &amp; 0 \leq x \leq 100 \ 1, &amp; 100 \leq x \end{cases})</td>
</tr>
</tbody>
</table>

The performance scores in the form of fuzzy sets are calculated using the Equation 1 and tabulated in table 2. The total performance scores of the three alternatives S1, S2, and S3 are shown in the last row of Table 2.

Table 2: The scores of technological advances derived from Table 1

<table>
<thead>
<tr>
<th>Item</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Turbo-shafts (kW)</td>
<td>1</td>
<td>0.41</td>
<td>0.46</td>
</tr>
<tr>
<td>2 Weight empty (kg)</td>
<td>0.25</td>
<td>0.72</td>
<td>0.84</td>
</tr>
<tr>
<td>3 Max level speed (km/h)</td>
<td>0.71</td>
<td>0.61</td>
<td>0.46</td>
</tr>
<tr>
<td>4 Max disc loading (kg/m^2)</td>
<td>0.49</td>
<td>0.67</td>
<td>0.25</td>
</tr>
<tr>
<td>5 Service ceiling (m)</td>
<td>0.6</td>
<td>0.8</td>
<td>0.09</td>
</tr>
<tr>
<td>6 Maximal range standard fuel (km)</td>
<td>0.3</td>
<td>0.41</td>
<td>0.54</td>
</tr>
<tr>
<td>7 Endurance with maximum fuel</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>8 g-limits</td>
<td>0.5</td>
<td>1</td>
<td>0.25</td>
</tr>
<tr>
<td>9 Mission capable</td>
<td>0.9</td>
<td>0.93</td>
<td>0.92</td>
</tr>
<tr>
<td>Total</td>
<td>5.25</td>
<td>6.55</td>
<td>4.31</td>
</tr>
</tbody>
</table>

Next, some of the evaluations given by an expert are in the form of linguistic terms. The linguistic terms with the corresponding membership function as defined in Figure 1 are used to facilitate the subjective assessment in evaluating the weapon systems.
Then, from this term the scores for the criteria are allocated by taking an appropriate mean of fuzzy number as tabulated in Table 2.

**Table 2: Linguistic value**

<table>
<thead>
<tr>
<th>Fuzzy language</th>
<th>The mean of fuzzy number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very good</td>
<td>1</td>
</tr>
<tr>
<td>Good</td>
<td>0.75</td>
</tr>
<tr>
<td>Fair</td>
<td>0.5</td>
</tr>
<tr>
<td>Poor</td>
<td>0.25</td>
</tr>
<tr>
<td>Very poor</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3 depicted the original data of performance scores given by the expert’s opinion on particular subcriteria in the form of linguistic evaluation.

**Table 3: Linguistic Logistic capability evaluation**

<table>
<thead>
<tr>
<th>Item</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Reliability</td>
<td>Fair</td>
<td>Good</td>
<td>Good</td>
</tr>
<tr>
<td>2 Maintenance ability</td>
<td>Very Good</td>
<td>Good</td>
<td>Good</td>
</tr>
<tr>
<td>3 Convey</td>
<td>Fair</td>
<td>Very Good</td>
<td>Good</td>
</tr>
<tr>
<td>4 Economics</td>
<td>Very good</td>
<td>Good</td>
<td>Very Good</td>
</tr>
<tr>
<td>5 Flexibility for selecting weapon</td>
<td>Good</td>
<td>Very Good</td>
<td>Good</td>
</tr>
</tbody>
</table>

Using an appropriate mean of fuzzy number in Table 2, the linguistic terms are translated as in Table 4.

**Table 4: The Score of Logistic capability evaluation**

<table>
<thead>
<tr>
<th>Item</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Reliability</td>
<td>0.5</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>2 Maintenance ability</td>
<td>1</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>3 Convey</td>
<td>Fair</td>
<td>1</td>
<td>0.75</td>
</tr>
<tr>
<td>4 Economics</td>
<td>1</td>
<td>0.75</td>
<td>1</td>
</tr>
</tbody>
</table>
The data of armament which comprised of sub-factors: Guns, anti-tank, missiles, and rockets are tabulated in Table 5. The performance scores of the sub-factors of armament for three attack helicopters are calculated and shown in Table 6.

Table 5: The data of armament for three attack helicopters

<table>
<thead>
<tr>
<th>Item</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>Membership function</th>
</tr>
</thead>
</table>
| Gun Caliber (mm)            | 30  | 30  | 20  | \( \mu_m = \begin{cases} 
0.5, & 20 \leq x \leq 30 \\
1, & x = 30 
\end{cases} \) |
| Feed                        | 30  | 30  | 20  | \( \mu_m = \begin{cases} 
0.5, & 20 \leq x \leq 30 \\
1, & x = 30 
\end{cases} \) |
| Firing rate (r/m)           | 900 | 625 | 650 | \( \mu_{fr} = \begin{cases} 
(x - 500)/500, & 500 \leq x \leq 1000 \\
1, & 1000 \leq x 
\end{cases} \) |
| Feed                        | 300 | 1200| 750 | \( \mu_{gf} = \begin{cases} 
(x - 200)/1000, & 200 \leq x \leq 1200 \\
1, & 1200 \leq x 
\end{cases} \) |
| Anti-tank Missiles Feed     | 5092| 4634|     | \( \mu_{af} = \begin{cases} 
x/16, & 0 \leq x \leq 16 \\
1, & 16 \leq x 
\end{cases} \) |
| Firing range (km)           | 5   | 8   | 8   | \( \mu_{at} = \begin{cases} 
x/10, & 0 \leq x \leq 10 \\
1, & 10 \leq x 
\end{cases} \) |
| Firing accuracy (%)         | 80  | 76  | 87  | \( \mu_{ia} = \begin{cases} 
x/100, & 0 \leq x \leq 100 \\
1, & 100 \leq x 
\end{cases} \) |
| Air-to-air missiles Feed    | 8   | 4   | 4   | \( \mu_{at} = \begin{cases} 
x/10, & 0 \leq x \leq 10 \\
1, & 10 \leq x 
\end{cases} \) |
| Firing accuracy (%)         | 85  | 90  | 50  | \( \mu_{ia} = \begin{cases} 
x/100, & 0 \leq x \leq 100 \\
1, & 100 \leq x 
\end{cases} \) |
| Rocket Feed                 | 20  | 4   | 4   | \( \mu_{rf} = \begin{cases} 
x/100, & 0 \leq x \leq 100 \\
1, & 100 \leq x 
\end{cases} \) |
| Caliber (mm)                | 70  | 70  | 70  | \( \mu_{rf} = \begin{cases} 
x/100, & 0 \leq x \leq 100 \\
1, & 100 \leq x 
\end{cases} \) |

Table 6: The Scores of armament for three attack helicopters

<table>
<thead>
<tr>
<th>Item</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gun Firing rate (r/m)</td>
<td>0.8</td>
<td>0.25</td>
<td>0.5</td>
</tr>
<tr>
<td>Feed</td>
<td>0.1</td>
<td>1</td>
<td>0.75</td>
</tr>
<tr>
<td>Anti-tank Missiles Firing range (km)</td>
<td>1</td>
<td>1</td>
<td>0.75</td>
</tr>
<tr>
<td>Firing accuracy (%)</td>
<td>0.5</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>Feed</td>
<td>0.8</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Firing accuracy (%)</td>
<td>0.85</td>
<td>0.9</td>
<td>0.5</td>
</tr>
<tr>
<td>Air-to-air missiles Firing accuracy (%)</td>
<td>0.8</td>
<td>0.76</td>
<td>0.76</td>
</tr>
<tr>
<td>Feed</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>Rocket Firing accuracy (%)</td>
<td>0.8</td>
<td>0.9</td>
<td>0.5</td>
</tr>
<tr>
<td>Feed</td>
<td>0.76</td>
<td>0.76</td>
<td></td>
</tr>
<tr>
<td>Caliber (mm)</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>Total</td>
<td>7.35</td>
<td>7.57</td>
<td>6.08</td>
</tr>
</tbody>
</table>
Lastly, the scores of the other criteria to select the best weapon that are Avionics, and Subsist ability are displayed in Table 7 and Table 8.

**Table 7: The Scores of Avionics**

<table>
<thead>
<tr>
<th>Item</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Pilot night vision system</td>
<td>0.5</td>
<td>1</td>
<td>0.25</td>
</tr>
<tr>
<td>2 Target acquisition and designation system</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>3 Integrate system</td>
<td>0.25</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>4 Global positioning system</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1.75</td>
<td>4</td>
<td>1.75</td>
</tr>
</tbody>
</table>

**Table 7: The Scores of Subsist Ability**

<table>
<thead>
<tr>
<th>Item</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Amor-protection</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>2 Counter-detected</td>
<td>0.75</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>3 Pilot-protested</td>
<td>1</td>
<td>0.75</td>
<td>0.5</td>
</tr>
<tr>
<td>4 Noise</td>
<td>1</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>5 NBS protection</td>
<td>1</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1.75</td>
<td>4</td>
<td>2.75</td>
</tr>
</tbody>
</table>

In this research, the synthetic decision \( r_{ij} \), which is single-criterion evaluation equivalent to \( B_{ij} \),

\[
r_{ij} = B_{ij}
\]  

where, \( B_{ij} \) is a fuzzy subset in \( P \); it can also be represented by a fuzzy vector,

\[
B_{ij} \in M_{1 \times J}
\]

In ensuring the total score remains in the range \([0, 1]\) the normalisation concept in Weon and Kim (2001) was adopted. The total of fuzzy marks of each criterion are used as input to develop the normalised synthetic score value. This is the seventh step of the subjective evaluation method. The calculation of normalised synthetic score value was based on the aggregation of multiplication score mark of each criterion and weight. The normalised synthetic score value is then calculated using Equation (4).

\[
\text{Normalised synthetic score} = \frac{1}{N} \sum_{mi} r_{mi}
\]

where \( N \) is the number of criteria in each factor.

### 2. THE FUZZY EVALUATION MODEL

The algorithm for the evaluation model consists of 4 steps as listed below:

| Step 1 | Calculate the normalised synthetic score value |
There are 4 steps in the proposed model for evaluating the best attack helicopters. The first step is to transform the input data into normalised synthetic score value. The proposed model then constructs the normalized synthetic score value as shown in Table 7. Each element in the table is calculated by using Equation (4), the total score of each criterion.

<table>
<thead>
<tr>
<th>Weapon Systems</th>
<th>Criteria 1</th>
<th>Criteria 2</th>
<th>Criteria 3</th>
<th>Criteria 4</th>
<th>Criteria 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0.332</td>
<td>0.313</td>
<td>0.350</td>
<td>0.233</td>
<td>0.400</td>
</tr>
<tr>
<td>S2</td>
<td>0.395</td>
<td>0.354</td>
<td>0.360</td>
<td>0.533</td>
<td>0.356</td>
</tr>
<tr>
<td>S3</td>
<td>0.273</td>
<td>0.333</td>
<td>0.290</td>
<td>0.233</td>
<td>0.244</td>
</tr>
</tbody>
</table>

Table 8 shows the fuzzy rules generated by the proposed model from the weapons system data in terms of rules properties, number of rules, maximum length, minimum length are 3, 3 and 3 respectively.

<table>
<thead>
<tr>
<th>Decision Criteria</th>
<th>Factor Rule</th>
<th>Linguistic Variable</th>
<th>Description</th>
<th>Appraisal Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>$C_2 \cap (C_3 \cup C_4)$</td>
<td>$A_1$</td>
<td>Satisfactory</td>
<td>$v$</td>
</tr>
<tr>
<td>C2</td>
<td>$C_3 \cap (C_2 \cup C_4)$</td>
<td>$A_1$</td>
<td>Satisfactory</td>
<td>$v$</td>
</tr>
<tr>
<td>C3</td>
<td>$C_4 \cap (C_2 \cup C_3)$</td>
<td>$A_1$</td>
<td>Satisfactory</td>
<td>$v$</td>
</tr>
</tbody>
</table>

Then the appraisal fuzzy value, $(d_j(m,l))$, of Table 10 is computed as follows (Othman et al., 2004d):

$$d_j(m,l) = 1 \land (1 - \tilde{c}(u_m) + A_k(v_l)),$$

where $j = 1, 2, 3$, $m = 1, 2, 3$, $l = 1, 2, \ldots, 11$ and $\tilde{c}(u_m)$ is the factor rule value.

The appraisal fuzzy values for decision criteria $DC_1$ are tabulated in Table 10.
Table 10: Appraisal Fuzzy Value for Decision Criteria DC\textsubscript{1}

<table>
<thead>
<tr>
<th>Appraisal Set</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.687</td>
<td>0.687</td>
<td>0.400</td>
<td>0.650</td>
</tr>
<tr>
<td>0.700</td>
<td>0.700</td>
<td>0.500</td>
<td>0.750</td>
</tr>
<tr>
<td>0.800</td>
<td>0.800</td>
<td>0.600</td>
<td>0.850</td>
</tr>
<tr>
<td>0.900</td>
<td>0.900</td>
<td>0.700</td>
<td>0.950</td>
</tr>
<tr>
<td>1.000</td>
<td>1.000</td>
<td>0.800</td>
<td>1.000</td>
</tr>
<tr>
<td>1.000</td>
<td>1.000</td>
<td>0.900</td>
<td>1.000</td>
</tr>
<tr>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Therefore, the appraisal product value \( D \) is calculated by multiplying all the elements of the appraisal fuzzy value, \( D_j \) obtained earlier. The formula is given in Equation 6.

\[
D = \left( \prod_{j=1}^{J} d_j(n,l) \right) = (\tilde{E}_1, \tilde{E}_2, \ldots, \tilde{E}_F, \ldots, \tilde{E}_L) \in M_{L \times 1} \tag{6}
\]

Assuming that \( E_{\alpha} \) is the \( \alpha \) level of \( \tilde{E}_n \), \( \alpha \in [0, 1] = I \), it should be noted that the sets \( E_{\alpha} \) are ordinary subsets of \( V \). Each \( E_{\alpha} \), \( H_1(E_{\alpha}) = \text{mid-point} \) is calculated. The calculated appraisal product value is shown in Table 11.

Table 11: Appraisal Product Value

<table>
<thead>
<tr>
<th>Appraisal Set</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.216</td>
<td>0.216</td>
<td>0.064</td>
<td>0.234</td>
</tr>
<tr>
<td>0.343</td>
<td>0.343</td>
<td>0.125</td>
<td>0.367</td>
</tr>
<tr>
<td>0.512</td>
<td>0.512</td>
<td>0.216</td>
<td>0.544</td>
</tr>
<tr>
<td>0.729</td>
<td>0.729</td>
<td>0.0343</td>
<td>0.769</td>
</tr>
<tr>
<td>1.000</td>
<td>1.000</td>
<td>0.512</td>
<td>1.000</td>
</tr>
<tr>
<td>1.000</td>
<td>1.000</td>
<td>0.729</td>
<td>1.000</td>
</tr>
<tr>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

The calculated values of the range of appraisal product value (\( \alpha \)), the different of range of appraisal product value (\( \Delta \alpha \)), and mean value of \( E_{\alpha} \), \( H_1(E_{\alpha}) \) are tabulated in Table 12.

Table 12: Calculated Range of \( \alpha \), \( \Delta \alpha \), and \( H_1(E_{\alpha}) \)

<table>
<thead>
<tr>
<th>( l )</th>
<th>Range ( \alpha )</th>
<th>( E_{\alpha} )</th>
<th>( H_1(E_{\alpha}) )</th>
<th>( \Delta \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>0.0000 &lt; ( \alpha ) ≤ 0.2160</td>
<td>{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}</td>
<td>0.50</td>
<td>0.2160</td>
</tr>
<tr>
<td>2.</td>
<td>0.2160 &lt; ( \alpha ) ≤ 0.3430</td>
<td>{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}</td>
<td>0.55</td>
<td>0.1270</td>
</tr>
<tr>
<td>3.</td>
<td>0.3430 &lt; ( \alpha ) ≤ 0.5120</td>
<td>{0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}</td>
<td>0.60</td>
<td>0.1690</td>
</tr>
<tr>
<td>4.</td>
<td>0.5120 &lt; ( \alpha ) ≤ 0.7290</td>
<td>{0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}</td>
<td>0.65</td>
<td>0.2170</td>
</tr>
<tr>
<td>5.</td>
<td>0.7290 &lt; ( \alpha ) ≤ 1.0000</td>
<td>{0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}</td>
<td>0.70</td>
<td>0.2710</td>
</tr>
<tr>
<td>6.</td>
<td>1.0000 &lt; ( \alpha ) ≤ 1.0000</td>
<td>{0.5, 0.6, 0.7, 0.8, 0.9, 1}</td>
<td>0.75</td>
<td>0.0000</td>
</tr>
<tr>
<td>7.</td>
<td>1.0000 &lt; ( \alpha ) ≤ 1.0000</td>
<td>{0.6, 0.7, 0.8, 0.9, 1}</td>
<td>0.80</td>
<td>0.0000</td>
</tr>
<tr>
<td>8.</td>
<td>1.0000 &lt; ( \alpha ) ≤ 1.0000</td>
<td>{0.7, 0.8, 0.9, 1}</td>
<td>0.85</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
The calculated values of the range of $\alpha$, $\Delta \alpha_i$, and $H_1(E_{\text{max}})$ are substituted in the following Equation (7) to calculate the satisfaction value in the final step of the method.

$$SV(m) = \frac{1}{\alpha_{\text{max}}} \sum_{i=1}^{11} H_1(E_{\text{max}}) \Delta \alpha_i$$  \hspace{1cm} (7)

where $\alpha$ = degree of appraisal product value, $\Delta \alpha_i = \alpha_i - \alpha_{i-1}$, $\alpha_0 = 0$, $H_1(E_{\text{max}})$ = mid-point $V_l$ ($l = 1, 2, 3, \ldots, 11$) and $\alpha_{\text{max}}$ = maximum degree of appraisal product value.

3. NUMERICAL RESULT

The results of evaluating the ranking of the best attack helicopters are tabulated in Table 13. Columns 2, 4 and 3, 5 of Table 13 illustrate the performance value and ranking order for measuring weapon system of attack helicopters alternatives S1, S2, and S3 respectively. The satisfaction values calculated by using the fuzzy evaluation model represent the performance values which are used to rank the attack helicopters alternatives. The satisfaction values in column 4 were 0.5683, 0.5947, 0.5622 and in column 5 the rankings were 2, 1, 3 respectively. The Cheng et al. (1999) method produced the performance values and ranking as listed in columns 2 and 3 as 0.151, 0.248, 0.143 and 2, 1, 3 respectively. Clearly, it shows that the satisfaction values are higher than the values obtained from Cheng et al.’s method. The higher value indicates that the reliable experts are satisfied with the attack helicopters alternatives offered. From these results, the fuzzy evaluation model shows outstanding performance when compared to Cheng et al.’s method with 100% accuracy in ranking three attack helicopters alternatives, S1, S2, and S3. Again the subjective evaluation method showed the advantage of simpler rules properties with a smaller number of rules and at minimum length.

<table>
<thead>
<tr>
<th>Method</th>
<th>Cheng et al.</th>
<th>Subjective evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Performance</td>
<td>Ranking</td>
</tr>
<tr>
<td>A</td>
<td>0.151</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>0.248</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>0.143</td>
<td>3</td>
</tr>
<tr>
<td>Acc %</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 13: Results of Attack Helicopters
3. CONCLUSION
A new fuzzy evaluation model has been proposed for the evaluation of the attack helicopters. Experimental results produced are comparable to results obtained from the model by Cheng et al. The significance of the research model was the application of a fuzzy expert system consisting of a set of rules in the form of IF (antecedent) THEN (Conclusion). The model could be used as an alternative approach in solving problems that involve uncertainties. The evaluation output would become more precise if the combination factors were accurately defined. The rule properties were also analysed to judge the strength of the subjective evaluation method. The results of the experiments showed remarkable ranking performance even with the use of small-sized rule properties.

References